

(6 pages) **Reg. No. :**

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**Code No. : 30012 E Sub. Code : GMMA 64/
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B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2020.

Sixth Semester

Mathematics/Mathematics with CA – Main

GRAPH THEORY

(For those who joined in July 2012 – 2015)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In any (p, q) graph $\sum d(v_i) = \underline{\hspace{2cm}}$.
(a) $2p$ (b) $2q$
(c) q (d) p
2. The complete graph with p points is denoted by $\underline{\hspace{2cm}}$.
(a) $K_{p,p}$ (b) K_p
(c) K_{p^2} (d) $K_{p,n}$

3. The length of the cycle C_4 is _____.
(a) 2 (b) 3
(c) 4 (d) 1
4. The partition $P = (6, 6, 5, 4, 3, 3, 1)$ is _____.
(a) not graphic (b) graphic
(c) not partition (d) none
5. Every tree is a _____.
(a) bipartite graph (b) cyclic graph
(c) acyclic graph (d) not connected graph
6. $A(p, q)$ graph G is a Tree if
(a) $p = q - 1$ (b) $q = p + 1$
(c) $p = q + 1$ (d) $q = p + 2$
7. $K_{3,3}$ is _____.
(a) planar graph (b) non-planar graph
(c) cyclic (d) path
8. The chromatic number of a tree is _____.
(a) 6 (b) 4
(c) 2 (d) 1

9. The coefficient of λ^{p-1} in $f(G, \lambda)$ is

(a) q (b) $-q$

(c) p (d) $-p$

10. The constant term is $f(G, \lambda)$ is

(a) 1 (b) 2

(c) 3 (d) 0

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If G is a K -regular bigraph with bipartition (V_1, V_2) and $K > 0$, then prove that $|V_1| = |V_2|$.

Or

(b) Prove that $r(m, n) = r(n, m)$.

12. (a) Prove that a graph G with P points and $\delta = \frac{p-1}{2}$ is connected.

Or

(b) Prove that if G is a K -connected graph then $q \geq \frac{pk}{2}$.

13. (a) If G is a graph in which the degree of every vertex is atleast two then prove that G contains a cycle.

Or

- (b) Prove that every Hamiltonian graph is 2-connected.
14. (a) If G is a plane connected (p, q) graph without triangles and $p \geq 3$ then prove that $q \leq 2p - 4$.

Or

- (b) If a (p_1, q_1) graph and a (p_2, q_2) graph are homeomorphic, then prove that $p_1 + q_2 = p_2 + q_1$.
15. (a) Prove that if G is a tree with n points ($n \geq 2$) then $f(G, \lambda) = \lambda(\lambda - 1)^{n-1}$.

Or

- (b) Show that $\lambda^4 - 3\lambda^3 + 3\lambda^2$ cannot be the chromatic polynomial of any graph.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that the maximum number of lines among all p -point graphs with no triangles is

$$\left\lfloor \frac{p^2}{4} \right\rfloor.$$

Or

- (b) (i) Define α' and β'
(ii) Prove that $\alpha' + \beta' = p$.
17. (a) Prove that a graph G with atleast two points is bipartite iff all its cycles are of even length.

Or

- (b) Prove that in any graph G , $K \leq \lambda \leq \delta$.
18. (a) State and prove Dirac theorem.

Or

- (b) Prove that $C(G)$ is well defined.
19. (a) State and prove Euler theorem.

Or

- (b) Show that $\psi'(K_n) = \begin{cases} n & \text{if } n \text{ is odd} \\ n-1 & \text{if } n \text{ is even} \end{cases}$

20. (a) State and prove Five colour theorem.

Or

(b) Prove that coefficients of $f(G, \lambda)$ is alternate in sign.
